7. $5.373737\ldots = 5 + 0.37 + 0.0037 + \cdots$

$$= 5 + \underbrace{\frac{37}{100} + \frac{37}{(100)^2} + \frac{37}{(100)^3} + \cdots}$$

geometric series with $a = 37/100$ and $r = 1/100$.

$$\therefore S_\infty = \frac{a}{1-r} = \frac{37/100}{1-1/100} = \frac{37}{99}.$$

$$\therefore 5.3737\ldots = 5 + \frac{37}{99} = \frac{532}{99}$$

10. Here $l = \pi$.

$$\therefore a_0 = \frac{1}{l}\int_c^{c+2l} f(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx.$$

$$a_n = \frac{1}{l}\int_c^{c+2l} f(x)\cos\frac{n\pi x}{l}\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x).\cos nx\,dx$$

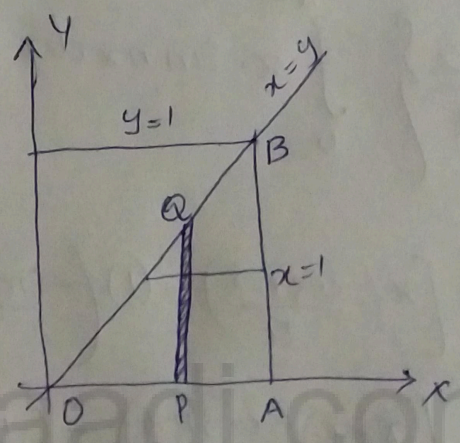
$$b_n = \frac{1}{l}\int_c^{c+2l} f(x)\sin\frac{n\pi x}{l}\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

16.a $$\int_0^1\int_y^1 \frac{x}{x^2+y^2}\,.dx.dy$$

$x = y \quad y = 0$

$x = 1 \quad y = 1$

OAB is the required region



Now changing order of integration.

At P, $y=0$ $\qquad x = 0$ to 1.

At Q, $y = x$

$$\therefore \int_0^1\int_0^x \frac{x}{x^2+y^2}\,dy\,dx = \int_0^1 x\cdot\frac{1}{x}\left[\tan^{-1}\frac{y}{x}\right]_0^x dx$$

$$= \int_0^1 \tan^{-1}1 - \tan^{-1}0\,dx = \int_0^1 \pi/2\,dx.$$

$$= \frac{\pi}{2}\int_0^1 dx = \frac{\pi}{2} //$$

20.b $\quad f(x) = \begin{cases} x, & 0<x<\pi/2 \\ \pi - x, & \pi/2<x<\pi. \end{cases}$

Half range Fourier Sine Series is given by

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}.$$

here $l = \pi$

$$b_n = \frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}\,dx$$

$$= \frac{2}{\pi}\int_0^\pi f(x)\sin nx\,dx.$$

$$= \frac{2}{\pi}\left\{\int_0^{\pi/2} x\cdot\sin nx\,dx + \int_{\pi/2}^{\pi}(\pi - x)\sin nx\,dx\right\}$$

$$= \frac{2}{\pi}\left\{\left[x\left(\frac{-\cos nx}{n}\right) - (1)\left(-\frac{\sin nx}{n^2}\right)\right]_0^{\pi/2} + \left[(\pi - x)\left(\frac{-\cos nx}{n}\right) - (-1)\left(-\frac{\sin nx}{n^2}\right)\right]_{\pi/2}^{\pi}\right\}$$

$$= \frac{2}{\pi}\left\{\left[-\frac{x\cos nx}{n} + \frac{\sin nx}{n^2}\right]_0^{\pi/2} + \left[-\frac{(\pi-x)\cos nx}{n} - \frac{\sin nx}{n^2}\right]_{\pi/2}^{\pi}\right\}$$

$$= \frac{2}{\pi}\left\{\left(\cancel{-\frac{\pi/2 \cos n\pi/2}{n}} + \frac{\sin n\pi/2}{n^2}\right) + \left(\cancel{\frac{\pi/2 \cdot \cos n\pi/2}{n}} + \frac{\sin n\pi/2}{n^2}\right)\right\}$$

$$= \frac{2}{\pi} \cdot \frac{2\sin n\pi/2}{n^2} = \frac{4}{\pi n^2} \sin n\pi/2 .$$

$$\therefore f(x) = \sum_{n=1}^{\infty} \frac{4}{\pi n^2} \sin \frac{n\pi}{2} \cdot \sin nx$$