Reg No   Name

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

First Semester B.Tech Degree Examination December 2021 (2019 scheme)

Course Code: MAT101

Course Name: LINEAR ALGEBRA AND CALCULUS

(2019 -Scheme)

Max. Marks: 100   Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks   Marks

1 Find the rank of the matrix $\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & -4 \\ 0 & 4 & 0 \end{bmatrix}$ (3)

2 Find the Eigen values of the matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$. What are the Eigen values of A^2, A^{-1} without using its characteristic equation. (3)

3 If $z = \frac{xy}{x^2+y^2}$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$. (3)

4 Show that the equation $u(x,t) = \sin(x - ct)$, satisfies wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$. (3)

5 Evaluate $\int_0^3 \int_0^2 \int_0^1 xyz\, dx\, dy\, dz$. (3)

6 Find the mass of the lamina with density $\delta(x,y) = x + 2y$ is bounded by the x −axis, the line $x = 1$ and the curve $y^2 = x$. (3)

7 Find the rational number represented by the repeating decimal 5.373737 ……… (3)

8 Examine the convergence of $\sum_{k=1}^{\infty} \frac{k^2}{2k^2+3}$ (3)

9 Find the Taylor series expansion of $f(x) = \sin\pi x$ about $x = \frac{1}{2}$ (3)

10 If $f(x)$ is a periodic function with period 2π defined in $[-\pi, \pi]$. Write the Euler's formulas a_0, a_n, b_n for $f(x)$. (3)

PART B

Answer one full question from each module, each question carries 14 marks.

MODULE 1

11 a Solve the following linear system of equations using Gauss elimination method (7)

$$x + 2y - z = 3$$
$$3x - y + 2z = 1$$
$$2x - 2y + 3z = 2$$

b Find the eigenvalues and eigenvectors of $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ (7)

12 a Solve the following linear system of equations using Gauss elimination method. (7)

$$2x - 2y + 4z = 0$$
$$-3x + 3y - 6z + 5w = 15$$
$$x - y + 2z = 0$$

b Find the matrix of transformation that diagonalize the matrix $A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$. (7)

Also write the diagonal matrix.

MODULE 2

13 a The length and width of a rectangle are measured with errors of at most 3% and 4% respectively. Use differentials to approximate the maximum percentage error in the calculated area. (7)

b Find the local linear approximation L of $f(x, y, z) = xyz$ at the point $P(1,2,3)$. (7) Compute the error in approximation f by L at the point $Q(1.001, 2.002, 3.003)$.

14 a If $w = f(P, Q, R)$ where $P = x - y$, $Q = y - z$, $R = z - x$ prove that (7)

$$\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0.$$

b Locate all relative extrema and saddle points of $f(x, y) = 4xy - x^4 - y^4$ (7)

MODULE 3

15 a Find the area bounded by the parabolas $y^2 = 4x$ and $x^2 = \frac{y}{2}$. (7)

b Evaluate $\int_{-2}^{2} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} e^{-(x^2+y^2)} dx\, dy$ using polar coordinates. (7)

16 a Evaluate $\int_0^1 \int_y^1 \frac{x}{x^2+y^2} dx\, dy$ by reversing the order of integration. (7)

b Use triple integral to find the volume of the solid within the cylinder $x^2 + y^2 = 9$ and between the planes $z = 1$ and $x + z = 5$ (7)

MODULE 4

17 a Test the convergence of (i) $\sum_{k=1}^{\infty} \frac{3k^3 - 2k^2 + 4}{k^7 - k^3 + 2}$ (ii) $\sum_{k=1}^{\infty} \frac{k^k}{k!}$ (7)

b Test whether the series $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{\sqrt{k+1}}$ is absolutely convergent or conditionally convergent (7)

18 a Test the convergence of the series $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} + \cdots\cdots\cdots$ (7)

b Test the convergence of (i) $\sum_{k=1}^{\infty} \left(\frac{k}{k+1}\right)^{k^2}$ (ii) $\sum_{k=1}^{\infty} \frac{7^k}{k!}$ (7)

MODULE 5

19 a Find the Fourier series expansion of $f(x) = x - x^2$ in the range $(-1, 1)$. (7)

b Obtain the half range Fourier cosine series of $f(x) = e^{-x}$ in $0 < x < 2$ (7)

20 a Find the Fourier series expansion of $f(x) = x^2$ in the interval $-\pi < x < \pi$. (7)

Hence show that $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}$

b Obtain the half range Fourier sine series of $f(x) = \begin{cases} x, & 0 < x < \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} < x < \pi \end{cases}$ (7)

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